Internal Letter

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SUBJECT: COLD START STUDY

Search-the-Sky Statistics

INTRODUCTION

A preliminary analysis of a cold start capability for GPS indicates the need for a brute force search-of-the-sky to acquire the first satellite. A brief look at various strategies also indicates that searching a single doppler frequency for all satellites is one way to minimize search time. In the pursuit of the analysis of a cold start strategy and subsequent receiver design it became apparent that more information would be needed on satellite orbits, LOS velocities, and visibilities.

Bill Weiser has written a computer program, described in the appendices, that will calculate visibility and LOS velocity for any stationary user position and time. We decided to use this program to investigate the statistics associated with search-the-sky acquisition.

A description of the various statistics compiled and the effect of the results on receiver design and search strategy will now be presented. For tutorial value, a polar view of the satellites in their orbits is shown in Figure 1.

STATISTICAL ANALYSIS

Data on 305 satellites visible above an elevation angle of +10° was obtained from 40 runs of the satellite visibility program (Appendix I). The runs were obtained from randomly chosen (uniform distribution) user positions and times. This data represents a statistical description of satellite position and LOS velocity for arbitrary user initial conditions.

Figure 2 is a scattergram of satellite position with respect to the user for all 40 arbitrary user positions. We conclude that satellites are randomly distributed in azimuth but their density decreases with increasing elevation angle. The dependence on elevation angle is related to the decrease in visible sky area (Appendix III). For a constant sky area, the satellites seem to be uniformly distributed. The right-hand scale

of the figure shows the percentage of total satellites visible from zenith to the indicated elevation angle.

Figure 3 is the cumulative distribution function for the number of satellites visible within a certain elevation (or zenith) angle. This curve yields important information about how many satellites can be observed within a cone about zenith. The percent visible was normalized to those above 10°. This choice represents a reasonable practical limit.

An average of 7.63 satellites are visible above 10° at an arbitrary location and time. From the curve we see that an average of 6.0 satellites are visible above 19°. To have an average of 4.0 satellites visible requires an elevation angle no greater than 35°. For design purposes, the worst case number of satellites visible above a certain elevation angle is the important criterion. Figures 4 and 5 are histograms showing the vertical cone width (Appendix III) required for keeping a minimum of four or six satellites visible at all times. Figure 4 shows that a cone width of 130° (25° elevation angle) was required so that at least four satellites are always visible. Figure 5 represents a more conservative design which keeps six satellites visible at all times and requires a cone width of 170° (5° elevation angle). Trial run #12 in the 40 runs indicates only four satellites above an elevation angle of +10° and seven between 0° and +10°. This brings up the, as yet, unresolved issue as to what constitutes a minimum practical elevation angle for a wide variety of users. (As a point of interest, run #12 corresponds to a location in the Black Sea just off the coast of the USSR at Odessa.)

A histogram of the LOS velocities for all 305 visible satellites is shown in Figure 6. Obvious conclusions are that the LOS velocities do not exceed ±850 meters/sec. and that the distribution is bi-modal. This distribution shows that the least likely place to find a satellite is near zero doppler. This result agrees with Figure 2 showing the least likely satellite locations are also near zenith (Since in most cases satellites near zenith exhibit small LOS velocities.)

To further investigate the LOS velocity characteristics near zero doppler, the histogram of Figure 7 was developed. The 40 data values correspond to the minimum doppler satellite in each of the runs. This result shows that at least one satellite in each run has an LOS velocity less than ±325 meters/sec. Similar plots showing the distributuion of closest doppler about the bi-modal peaks are shown in Figures 8 and 9. For these plots the LOS velocity is spread over approximately ±190 meters/second which indiates a stronger central tendency than Figure 7. Since the doppler spread is less about the bi-modal peaks, they are logical starting points for a search-the-sky mode.

Figure 10 is a histogram of the absolute magnitude of the LOS velocity closest to a bi-modal peak for each of the 40 trial runs. This plot is intended to illustrate the narrow spread of LOS velocities if both positive and negative peaks are searched.

IMPACT ON RECEIVER DESIGN

The analysis results to date indicate important effects on the antenna design, doppler/code search strategy, satellite selection strategy, and receiver predetection noise bandwidth.

The antenna directive gain or beam power pattern must be broad enough to cover at least four satellites. From Figure 3 we see that, on the average, a cone at least 110° wide is needed for four satellites and one at least 142° wide is needed for six satellites. For a worst case coverage of four satellites (Figure 4), the cone of coverage must be at least 130° wide. A large cone of coverage and useable gains to within 5° of the horizon are essential features needed for the GPS antenna.

The doppler/code search and satellite selection strategies can be designed and tested using the search-the-sky statistics. Preliminary results indicate that to acquire the first satellite in minimum time it would be best to search all 24 satellites at some nominal range of doppler. Work in this area is continuing in an effort to optimize satellite selection for minimum acquisition time.

Assuming a minimum-doppler search strategy, it is possible to use Figures 8 and 9 to determine a receiver predetection noise bandwidth that will cover all the doppler necessary to acquire at least one satellite. For worst case LOS velocities of ±190 M/S, this requires a single frequency window of 1997 Hz centered about either +2233 or -2233 Hz. Since the distribution of Figure 9 has a strong central tendency, it may be better to narrow the search window and do multiple scans. For a search strategy involving both positive and negative LOS velocity peaks, Figure 10 indicates a receiver bandwidth of 1200 Hz is needed.

SUMMARY

1. The antenna cone of coverage must be at least 170° wide to guarantee a minimum of 6 visible satellites.

2. An average of at least 6 visible satellites requires a cone 142° wide.

3. The most probable LOS satellite velocities are ± 425 meters/second.

5. Most satellites are positioned at low elevation angles with nearly one-fifth occuring within 10° of the horizon.

6. A receiver predetection noise bandwidth of 1-2 KHz is adequate for a search-the-sky algorithm.
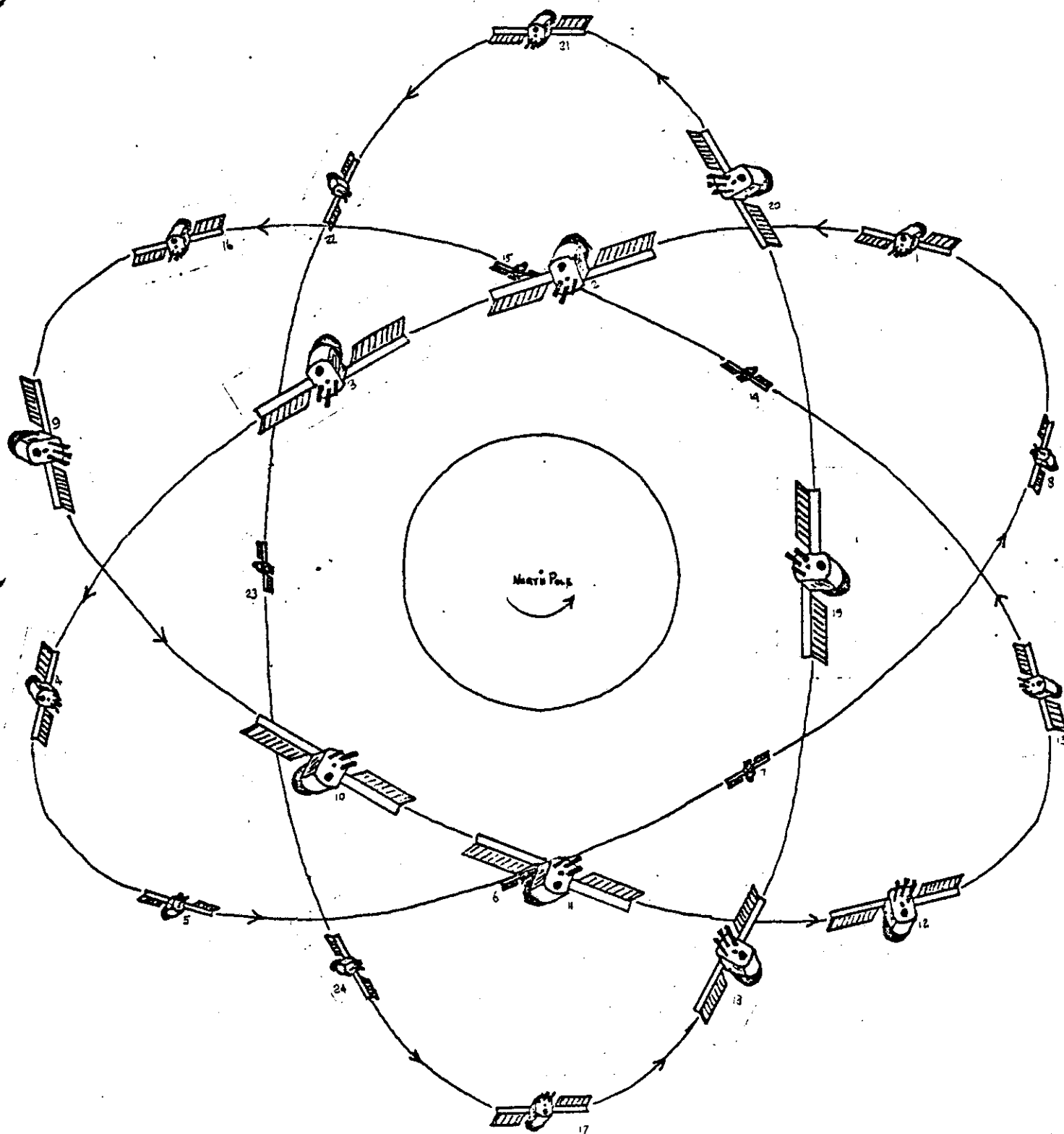
FIGURES

FIGURE 1.   A Polar View of the twenty-four GPS Satellites
in their respective orbital positions.

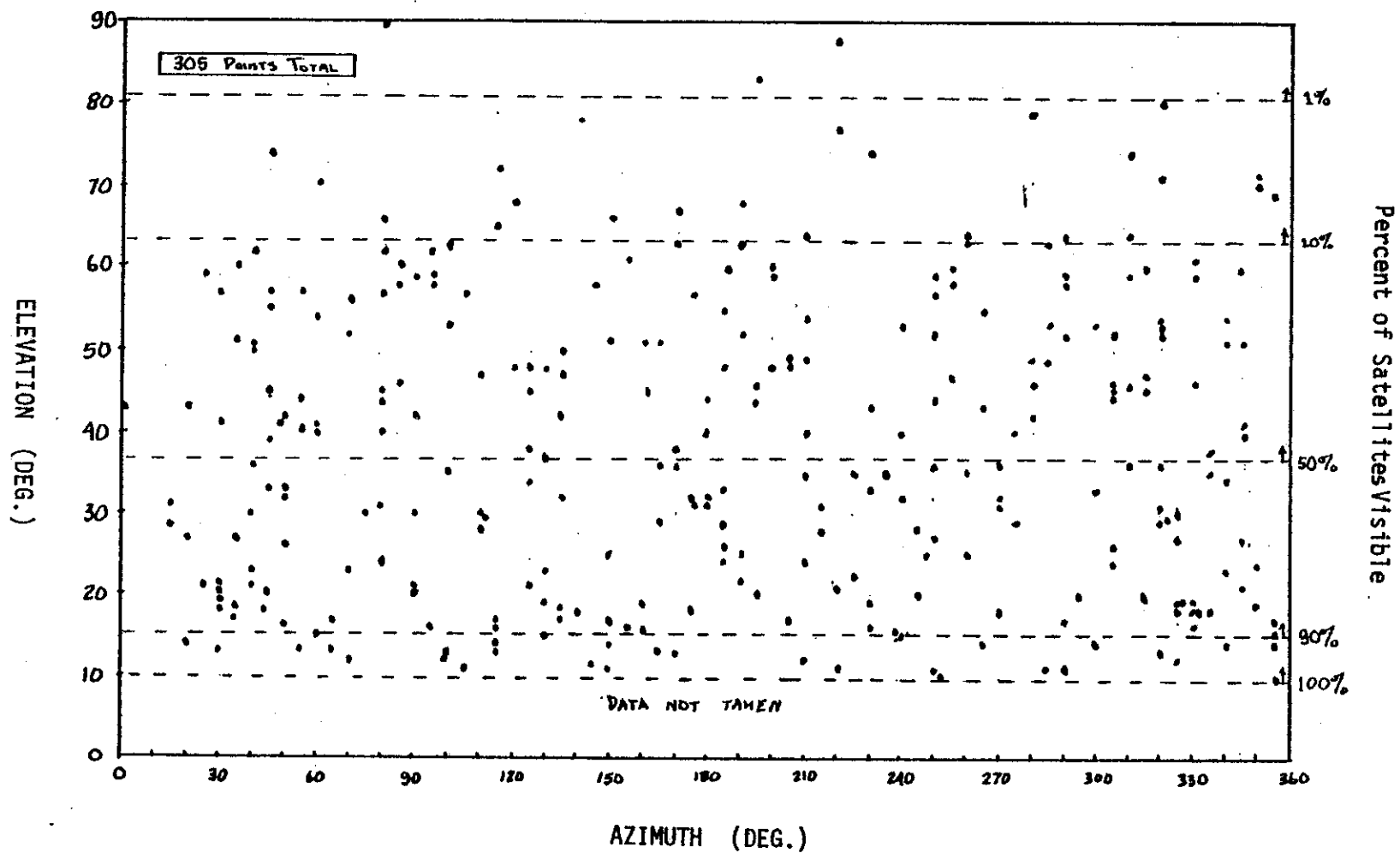FIGURE: 2.   SCATTERGRAM OF SATELLITE POSITIONS
WITH RESPECT TO AN ARBITRARY USER.
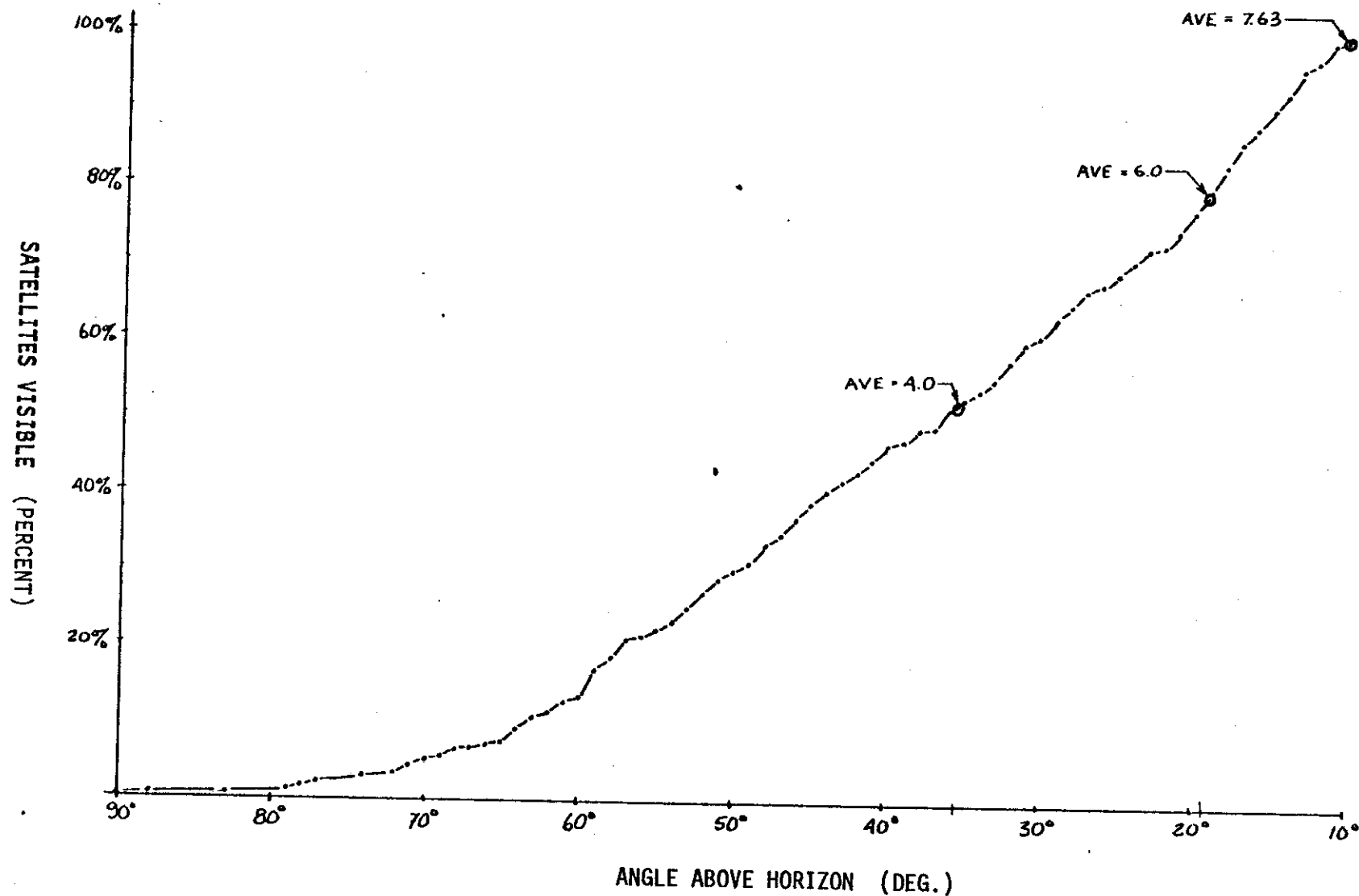
FIGURE: 3. CUMULATIVE DISTRIBUTION FUNCTION FOR SATELLITES VISIBLE AS A FUNCTION OF ELEVATION ANGLE.

Verticle Cone Size (Degrees)
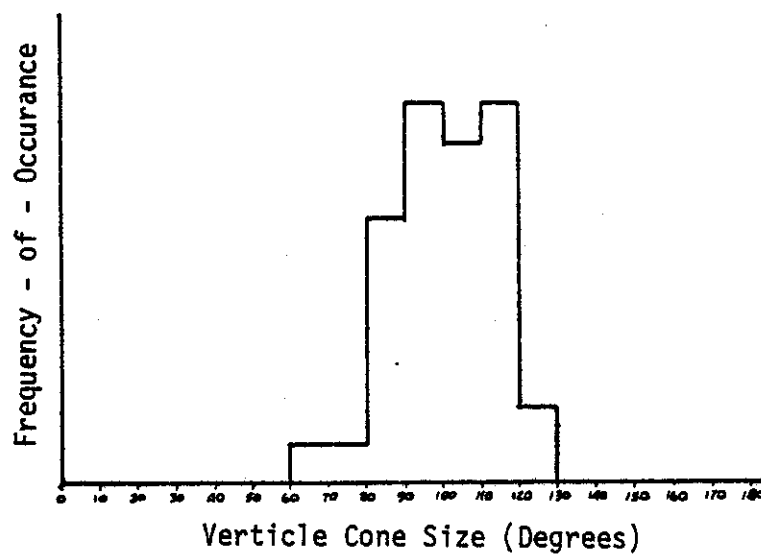
FIGURE: 4. HISTOGRAM OF VERTICAL CONE SIZE REQUIRED TO VIEW FOUR SATELLITES.

Verticle Cone Size (Degrees)
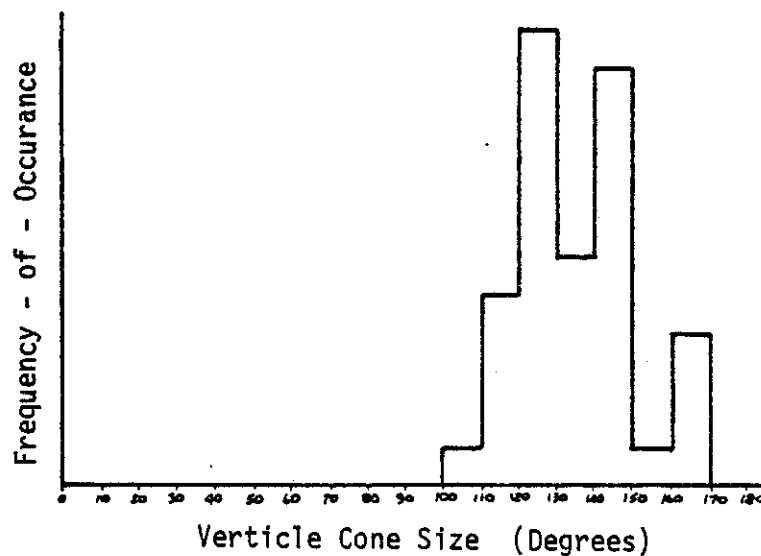
FIGURE: 5. HISTOGRAM OF VERTICAL CONE SIZE REQUIRED TO VIEW SIX SATELLITES.
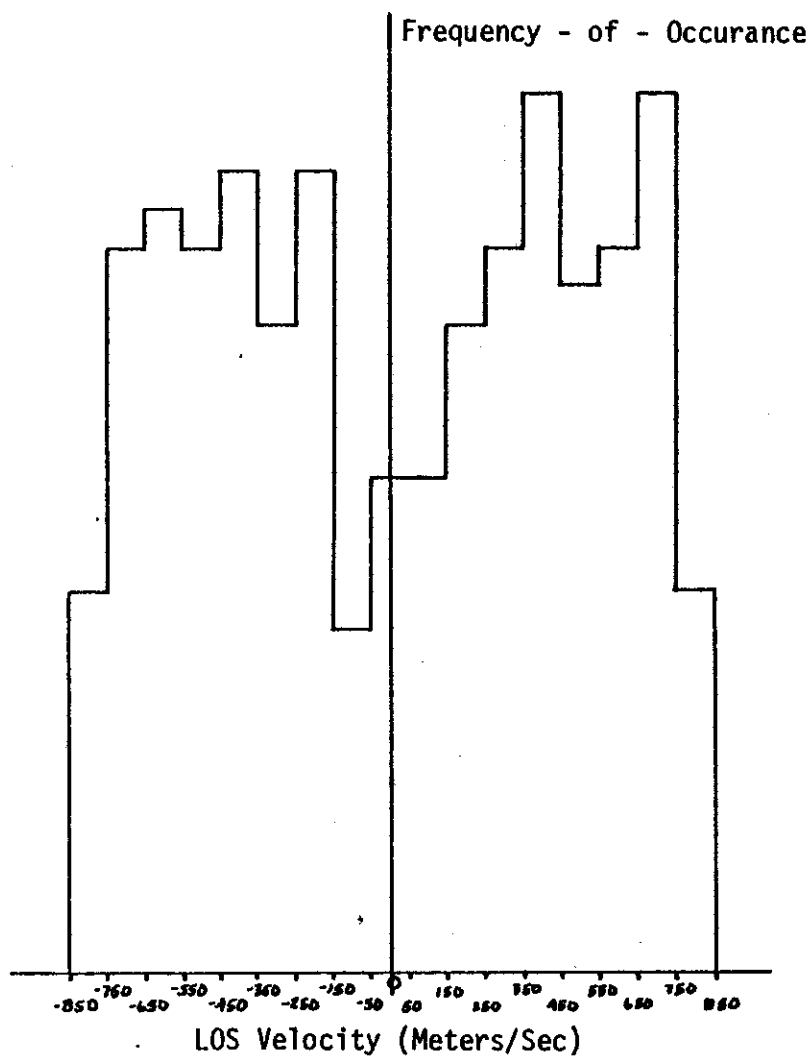
FIGURE: 6.   HISTOGRAM OF LOS VELOCITIES
              FOR ALL VISIBLE SATELLITES.
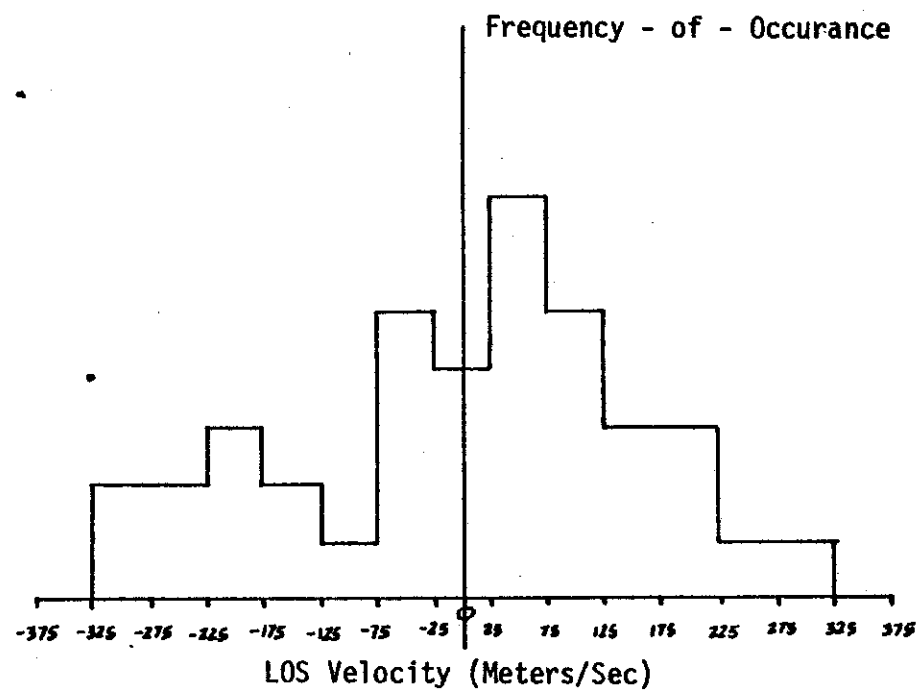
FIGURE: 7.   HISTOGRAM OF THE FORTY SMALLEST LOS VELOCITIES
              OVER THE ENSEMBLE OF TRIAL RUNS.

Frequency - of - Occurance

LOS Velocity (Meters/Sec)
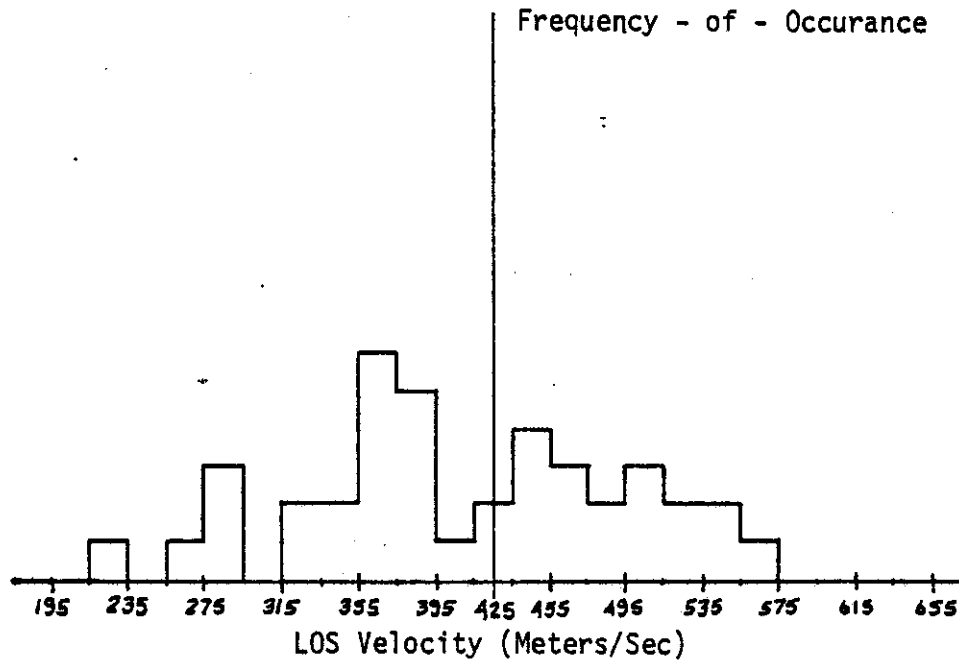
FIGURE: 8.   HISTOGRAM OF THE FORTY LOS VELOCITIES
              CLOSEST TO THE POSITIVE BI-MODAL PEAK.

Frequency -. of - Occurance

LOS Velocity (Meters/Sec)
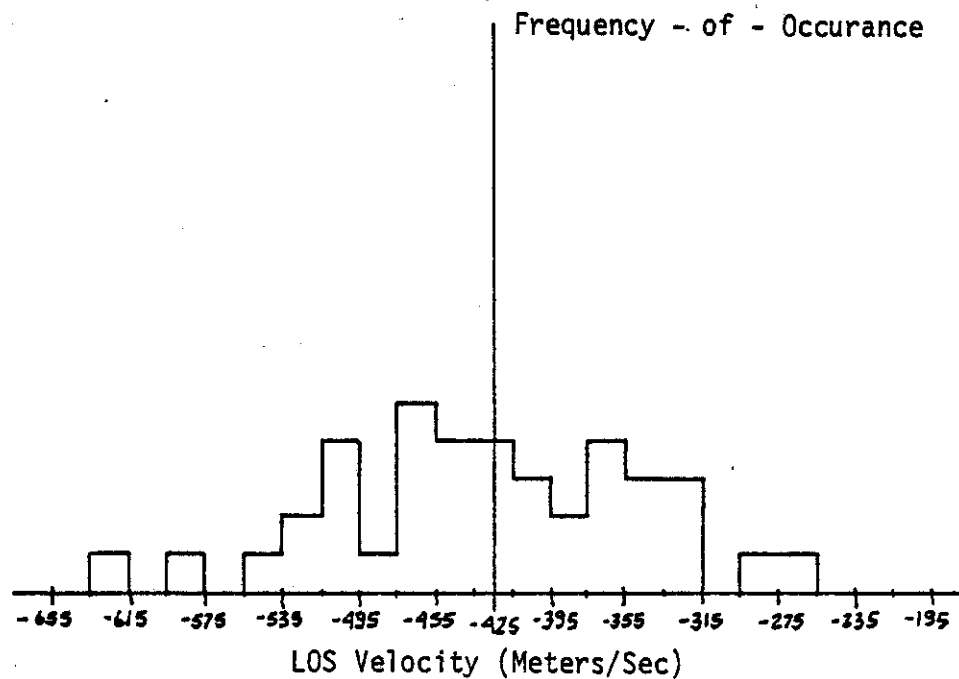
FIGURE: 9.   HISTOGRAM OF THE FORTY LOS VELOCITIES
              CLOSEST TO THE NEGATIVE BI-MODAL PEAK.
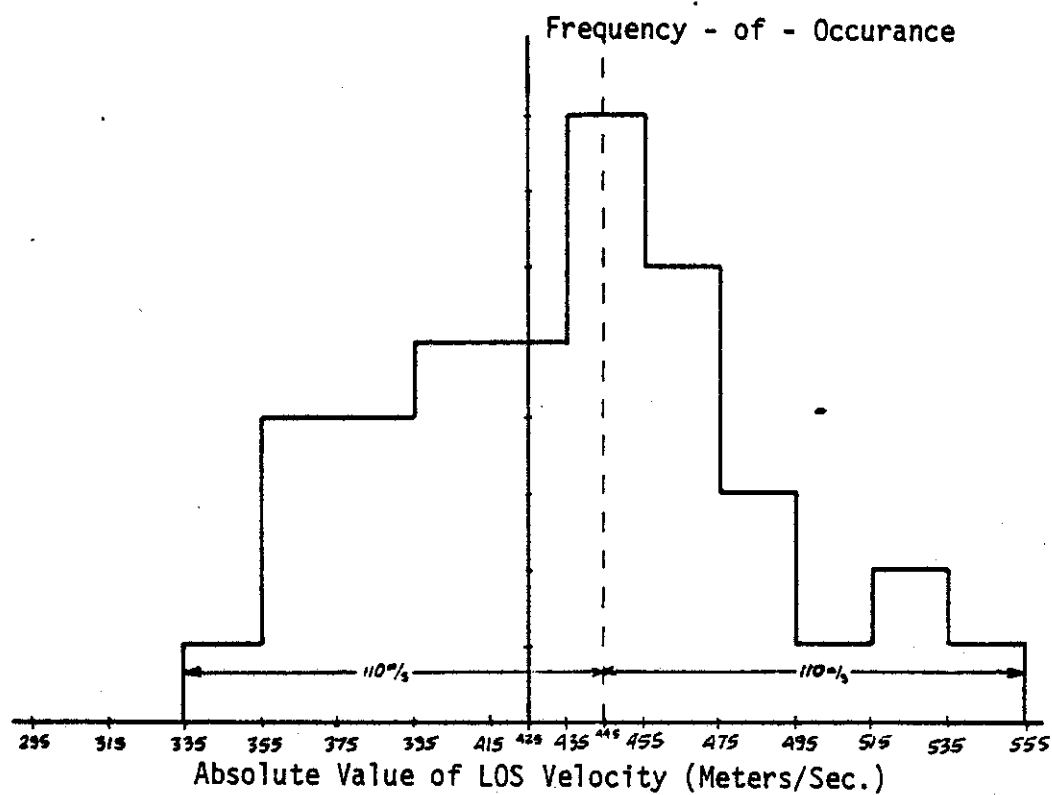
FIGURE: 10.   HISTOGRAM OF THE FORTY LOS VELOCITIES CLOSEST TO
EITHER THE POSITIVE OR THE NEGATIVE BI-MODAL PEAK.

APPENDICES

```
@MSG A GPS*SATELLITE
READY
>@XQT GPS*SATELLITE VISIBLE/VER-2

            ANOTHER RUN? (YES, NO)
>YES
        ***YES***

        SUPPLY: DAY, HOUR, MIN., SEC.,
        LONGITUDE (DEG, MIN, SEC), LATITUDE (DEG, MIN, SEC),
        ALTITUDE (FT).
>204, 12, 0, 0, -32, 0, 0, 45, 0, 0, 30000


        DO YOU WANT TO ADD ANY SV DATA? (YES, NO)
>NO

        ***NO ***

        SPECIFY THE ANGLE OF VISIBILITY
        0 TO 90 DEG ABOVE THE HORIZON

>5

                    SATELLITE VISIBILITY PROGRAM
                    **************************************


                    THIS PROGRAM WILL LIST THE VISIBLE GPS SATELLITES
                GIVEN A LOCATION AND GMT TIME.    IF  THE  SATELLITE IS
                VISIBLE (DEFINED TO BE   5.0  DEGREES ABOVE THE
                HORIZON) THEN THE AZIMUTH, ELEVATION ANGLES AND THE LOS
                (LINE OF SIGHT VELOCITY) WILL BE LISTED.
```

DAY	GMT TIME	LONGITUDE (D:M:S)	LATITUDE (D:M:S)	ALTITUDE (FT)
204	12:00:00	-32:00:00	45:00:00	30000

SATELLITE NUMBER	AZIMUTH (DEG)	ELEVATION (DEG)	LOS VELOCITY (M/S)
1	48.9	8.8	663.039
2	****	NOT VISIBLE	****
3	****	NOT VISIBLE	****
4	****	NOT VISIBLE	****
5	****	NOT VISIBLE	****
6	235.6	7.8	-640.850
7	251.0	58.7	-289.686
8	33.3	59.2	354.950
9	****	NOT VISIBLE	****
10	****	NOT VISIBLE	****
11	****	NOT VISIBLE	****
12	****	NOT VISIBLE	****
13	****	NOT VISIBLE	****
14	****	NOT VISIBLE	****
15	****	NOT VISIBLE	****
16	47.3	8.8	-263.166
17	307.1	59.4	-290.867
18	174.7	56.9	442.156
19	156.2	7.7	772.269
20	****	NOT VISIBLE	****
21	****	NOT VISIBLE	****
22	****	NOT VISIBLE	****
23	****	NOT VISIBLE	****
24	327.5	9.6	-725.377

```
            ANOTHER RUN? (YES, NO)
>NO
        ***NO ***

>@FIN
```

APPENDIX II.

SATELLITE VISIBILITY PROGRAM DESCRIPTION

(GPS * SATELLITE.VISIBLE/VER-2)

This program has evolved from GPS * SATELLITE.NTSG (a single satellite tracking program) and from GPS * SATELLITE.GDOP (a multi-satellite GDOP calculation program). The satellite position calculation section from the GDOP program was used since the satellite ephemeris parameters are in array form allowing multiple satellite position calculations.

In the .VISIBLE/VER-2 program, this position calculation section is in a DO-LOOP which steps through all twenty-four satellites. A write statement also in the DO-LOOP prints the location and LOS velocity information for each satellite vehicle.

Satellite ephemeris data is internally generated to simulate ideal conditions, i.e. Planar orbits, equal spacing, etc. One may insert actual SV ephemeris parameters into the program via the Namelist I/O Section.

The ephemeris data required for each satellite is:

TOA(*)	Almanac GPS Data Reference Time	(seconds)
DYC(*)	Day of Year Collected	(days)
TC(*)	Time Collected	(seconds)
MØ(*)	Mean Anomaly	(radians)
ECC(*)	Eccentricity	(meters/meters)
A(*)	Semi-Major Axis	(meters)
OMEGØ(*)	Right Ascension Angle	(radians)
INCL(*)	Inclination Angle	(radians)
ARGP(*)	Argument of Perigee	(radians)
OMEGD(*)	Rate of right ascension	(radians/second)

    *   AN INTEGER INDICATING THE SV NUMBER (1 to 24)

This data is entered via Fortran's Name List I/O.

The following input cards (or lines) would modify satellite number two's ephmeris data.

ᵬ$SVDATAᵬTOA(2)=dp,DYC(2)=integer,TC(2)=dp,
ᵬMO(2)=dp,ECC(2)=dp,A(2)=dp,OMEGO=dp,
ᵬINCL(2)=dp,ARGP(2)=dp,OMEGD(2)=dpᵬ$END

Notes: 1) "dp" indicates a double precision constant e.g. π: .3141592653590D1
      2) The ordering of the expressions is irrelevant.
      3) Any parameters not specified defaults to model values.
      4) See Fortran Manual for shortcuts for Namelist inputs.

If satellite data is modified, a listing of all satellite ephemeris data is produced for verification of your modifications.

The flow diagram on the next page summarizes the .VISIBLE/VER-2 program.

Flow Diagram of Satellite Visibility Program Version-2

ENTER

Initialization
Generate SV ephemeris data

```
        ANOTHER
         RUN        ──NO──>    END
          ?
         YES
          │
          ▼
   Prompt
   READ user Time, location data
          │
          ▼
        ADD
       SV DATA    ──YES──>   READ SV DATA
          ?                  WRITE SV DATA
         NO
          │
          ▼
```

Prompt, READ ANGLE OF VISIBILITY ABOVE HORIZON
PRINT HEADING

CALCULATE EARTH CENTERED EARTH FIXED (ECEF)
LOCATION VECTOR OF USER

GENERATE ECEF TO LOCAL TANGENT PLANE
COORDINATE SYSTEM CONVERSION MATRIX

DO FOR SV# = 1,24
    CALCULATE TIME SINCE REFERENCE TIME
END DO LOOP

DO FOR SV# = 1,24
1. CALC. ECEF POSITION & VELOCITY
    VECTOR OF SATELLITE.
2. CALC. LINE OF SIGHT (LOS) VECTOR
    BETWEEN SV & USER IN ECEF COORD.
3. NORMALIZE LOS POSITION VECTOR
4. CALC. DOPPLER FROM LOS VELOCITY VECT.
5. CONVERT LOS VECTOR TO THE LOCAL
    COORDINATE SYSTEM VECTOR

```
          YES     SV       NO
       ┌───────VISIBLE───────┐
       │          ?          │
       ▼                     ▼
```

1. CALC. LOCAL AZIMUTH
    AND ELEVATION ANGLES

2. CONVERT FROM RADIANS
    TO DEGREES
3. WRITE: Az, EL., DOPPLER.

PRINT:
'NOT VISIBLE'

END DO LOOP

B. WEISER
6/30/78

APPENDIX III.

DERIVATION OF SKY AREA AS A FUNCTION OF ELEVATION ANGLE

THE TOTAL VISIBLE AREA OF THE SKY IS A HALF-SPHERE WHOSE AREA IS:

$$A = \frac{4\pi R^2}{2} = 2\pi R^2$$

WHERE $4\pi R^2$ IS THE SURFACE AREA OF A SPHERE OF RADIUS 'R'.

DEFINE 'ϕ' AS THE <u>ELEVATION ANGLE</u> AND *NOTE THE* VERTICLE VISIBILITY CONE WHICH IS DEFINED IN FIGURE 'A'.

RECALL THAT IN SPHERICAL COORDINATES THAT VOLUME IS GIVEN BY:

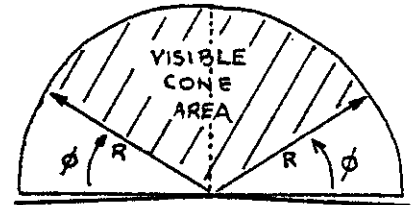$$V = \iiint \rho^2 \sin\psi \, d\rho \, d\theta \, d\psi$$

FIGURE A.

WHERE THE VARIABLES ARE DEFINED IN FIGURE B.

TO FIND SURFACE AREA WE CAN FIX ρ TO A CONSTANT RADIUS .SAY 'R' THE VOLUME INTEGRAL THEN REDUCES TO THE DOUBLE INTEGRAL:

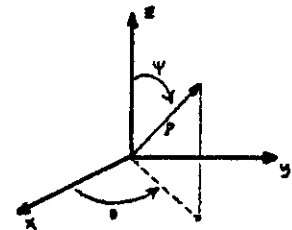$$S = \iint R^2 \sin\psi \, d\theta \, d\psi$$

FIGURE B.

THE LIMITS OF INTEGRATION ARE AS FOLLOWS:

θ : 0 to 2π

ψ : 0 to $\pi/2 - \phi$

EVALUATING THE SURFACE AREA INTEGRAL:

$$A = R^2 \int_0^{\pi/2-\phi} \int_0^{2\pi} \sin\psi \, d\theta \, d\psi$$

$$A = 2\pi R^2 \left[-\cos(\pi/2 - \phi) - -\cos(0) \right] = 2\pi R^2 \left[1 - \cos(\pi/2 - \phi) \right]$$

BUT $\cos(\pi/2 - \phi) = \sin(\phi)$

$$A = 2\pi R^2 \left(1 - \sin(\phi) \right)$$

THE PERCENAGE OF SKY VISIBLE IS GIVEN BY *THE* RATIO OF VISIBLE SKY AREA TO THE TOTAL SKY AREA TIMES 100%.

$$\boxed{P_{\text{ERCENT OF VISIBLE SKY}} = (1 - \sin(\phi)) \cdot 100\%}$$

BY DEFINING A <u>ZENITH ANGLE</u> ψ_0 SUCH THAT $\psi_0 = \pi/2 - \phi$ THE EQUATION BECOMES:

$$\boxed{P_{\text{ERCENT OF VISIBLE SKY}} = (1 - \cos(\psi_0)) \cdot 100\%}$$

A3-1

B. Weiser 7/7/